Cellular automata
Individual-based modeling

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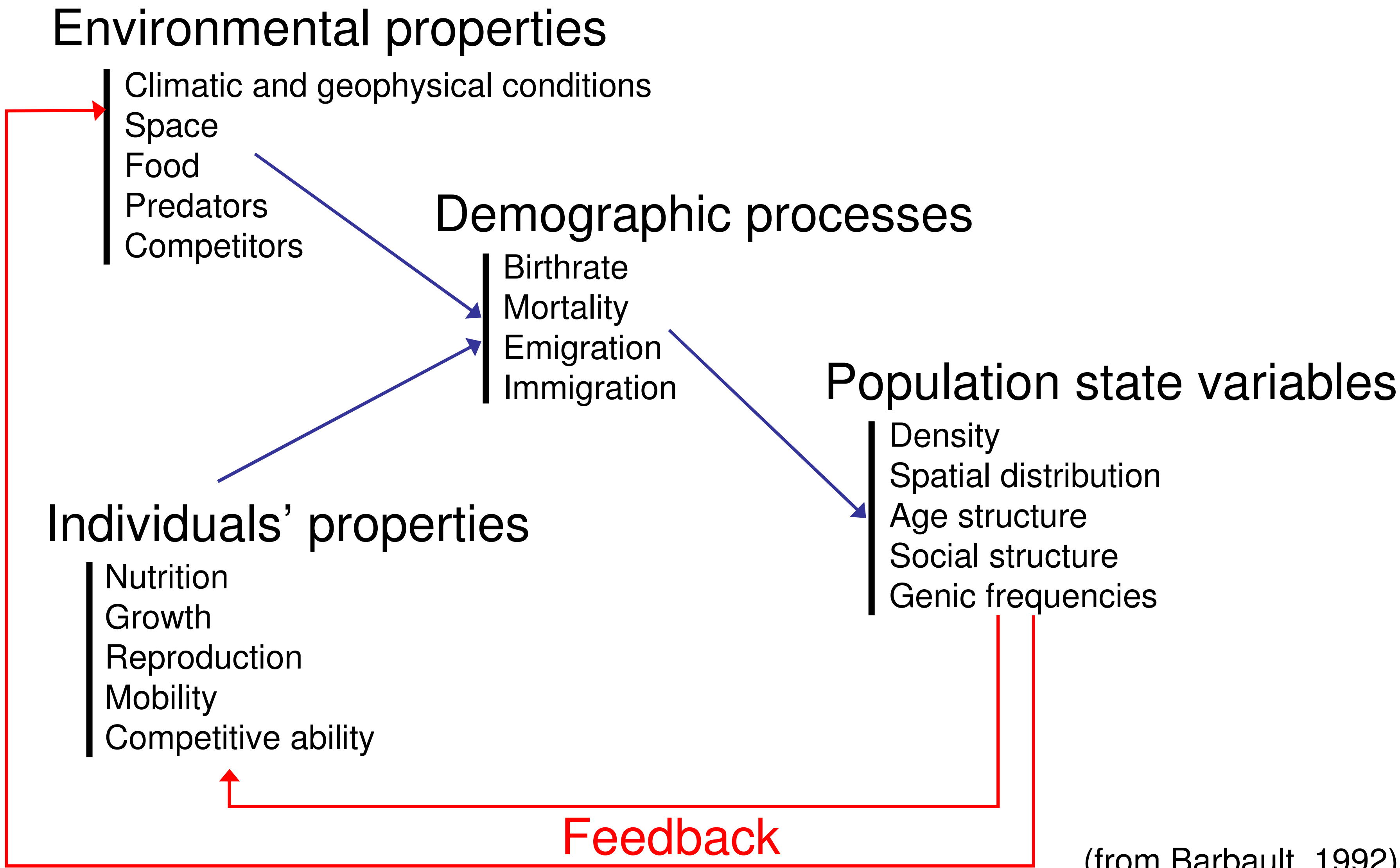
Individuals/Environment/Population



(from Barbault, 1992)

Mathematical models of population dynamics

Mean field models

- The state variable is one number
- Density, frequency, biomass, etc.

Age or stage structured models

- The state variable is a vector (matrix models)
- Density in each age class (Leslie) or life-stages (Lefkovich)

Individual based models

- The state variable is a vector of individual states

Approaches of complex systems

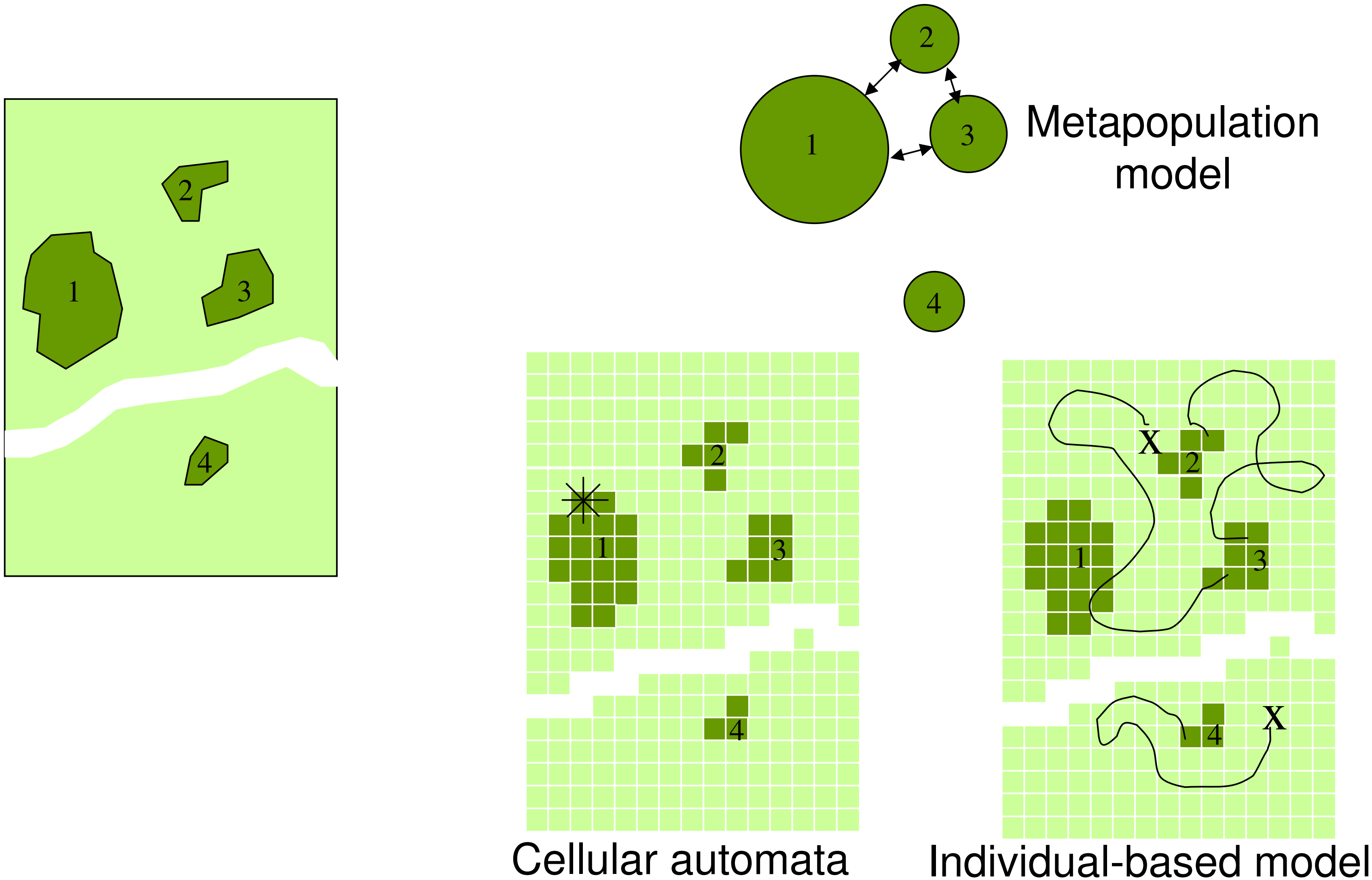
- **Analytical**
  element by element
  (neo-classical economy, plot, individual, etc.)
- **Holistic** or **systemic**
  global behaviour of the system
  (macro-economy, statistics)
- **Constructivist**
  articulation between individual behaviours of the elements (local) and the global behaviour of the system (global)

Mathematical vs. simulation-based approach

Criteria	Analytical	Simulation-based
Spatial environment	homogeneous	heterogeneous
Demographic stochasticity	unimportant	important
Rare events	unimportant	important
Biological and/or environmental discontinuities	unimportant	important
Number of individuals	large	small
Biological complexity	simple	complex

(from Gross et al., 1992)

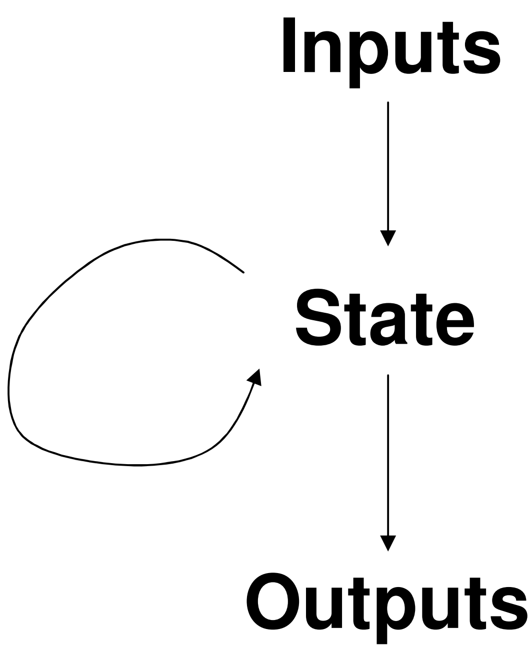
Spatially-explicit & individual-based simulation of population dynamics



Cellular automata

Automaton

- A set of inputs
- A set of states
- A transition function
- A set of outputs

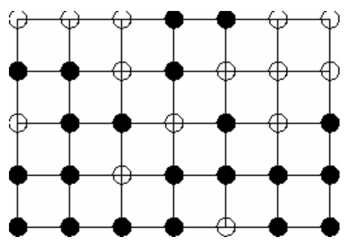
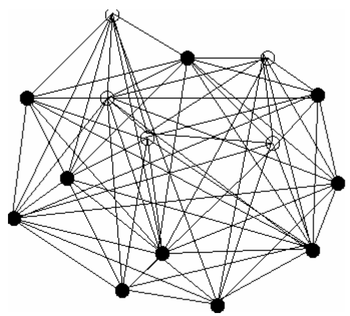
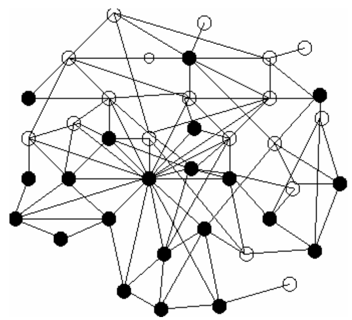
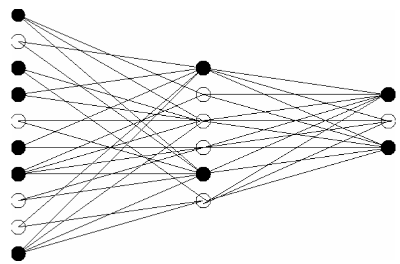


T	Input	State	Output
0	Nothing	Waiting	Menu
1	Ask coffee	Waiting	Menu
2	Nothing	Need = 2	Menu
3	Nothing	Need = 2	Need 2€
4	1€ coin	Need = 2	Need 2€
5	Nothing	Need = 1	Need 2€
6	Nothing	Need = 1	Need 1€
7	1€ coin	Need = 1	Need 1€
8	Nothing	Need = 0	Need 1€
9	Nothing	Waiting	Coffee
10	Nothing	Waiting	Menu

Network of automata

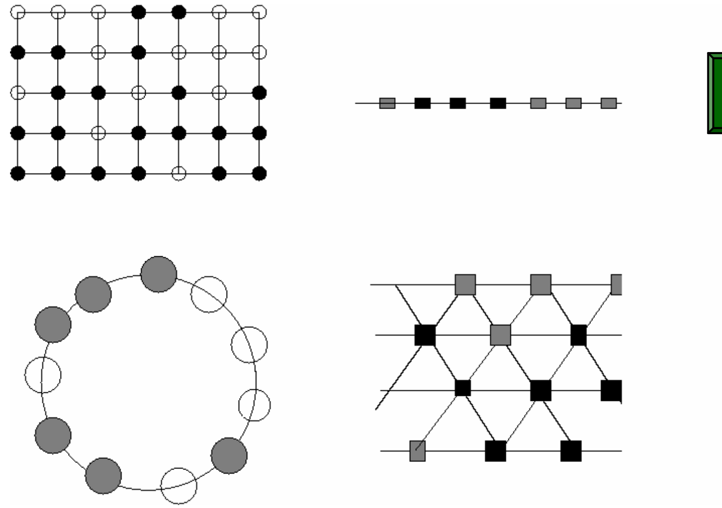
- A group of automata, the inputs for some are outputs for others

- Architecture: regular, total connectivity, random, layered

Cellular Automata

✓ Regular architecture

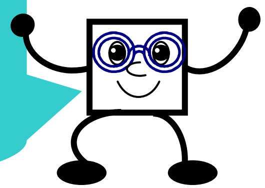


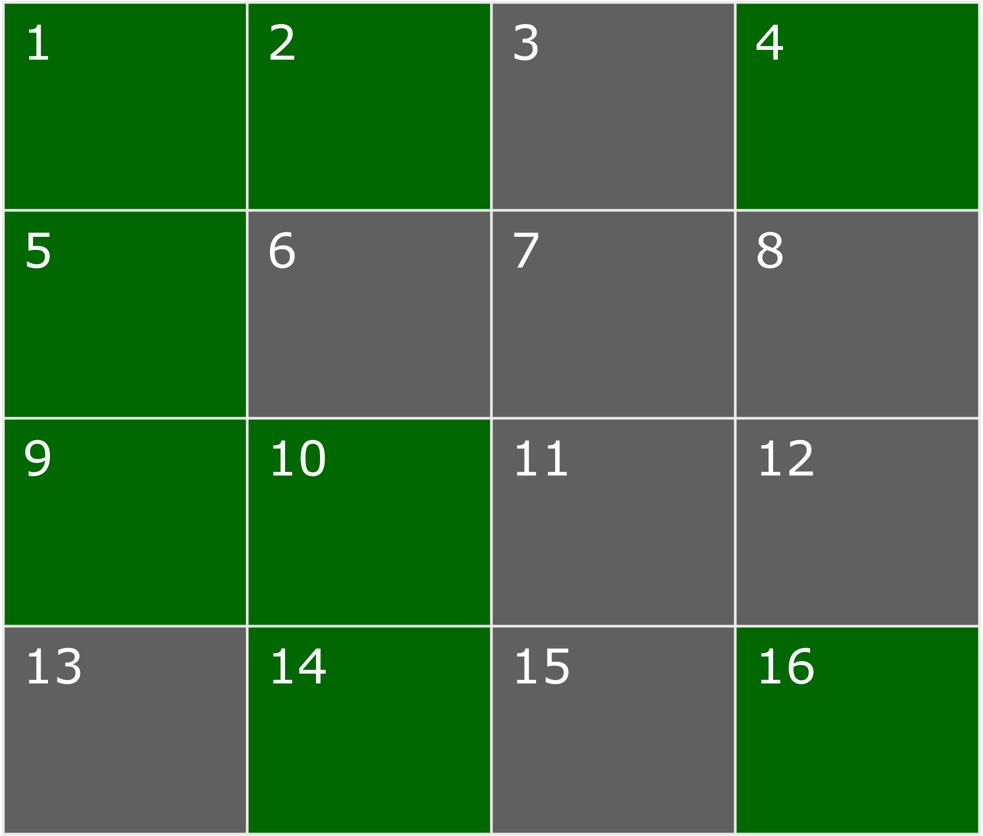
✓ Uniform and discrete transition function

✓ Synchronous and deterministic functionning

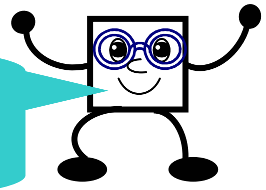
The Game of Life

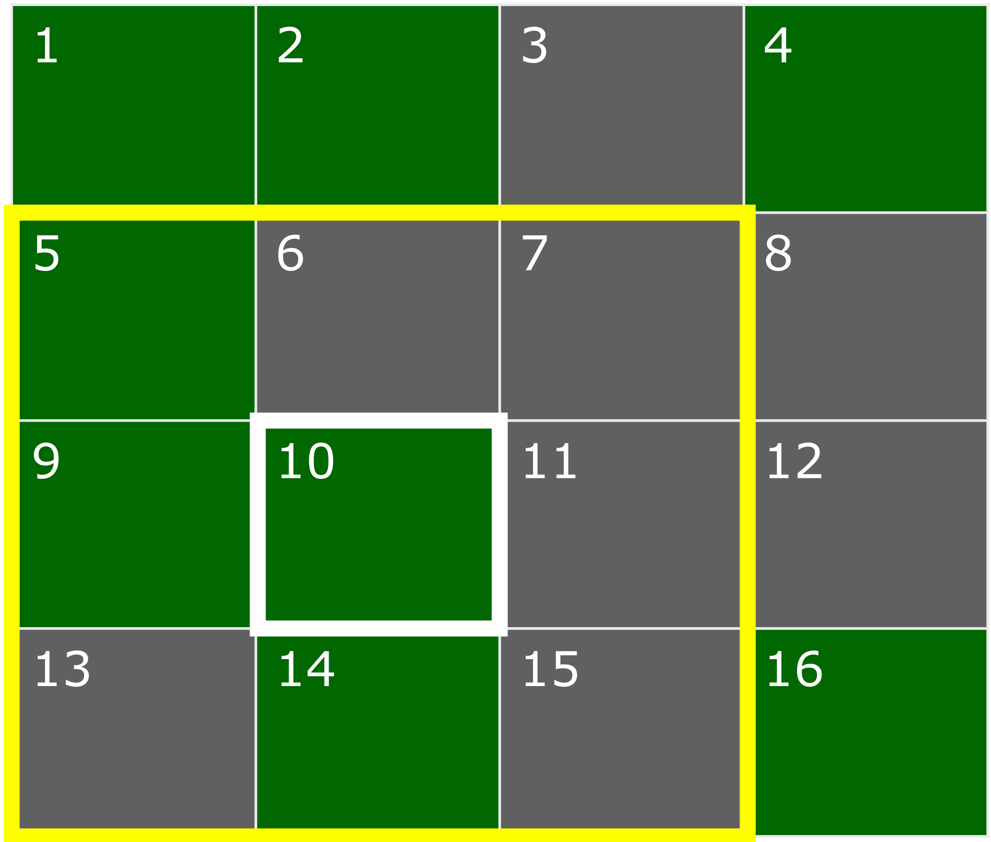
In the *Game of Life* proposed by John Conway,
cells states (alive -> green; dead -> grey)
are changing according to their own state
and the states of their 8 neighboring cells



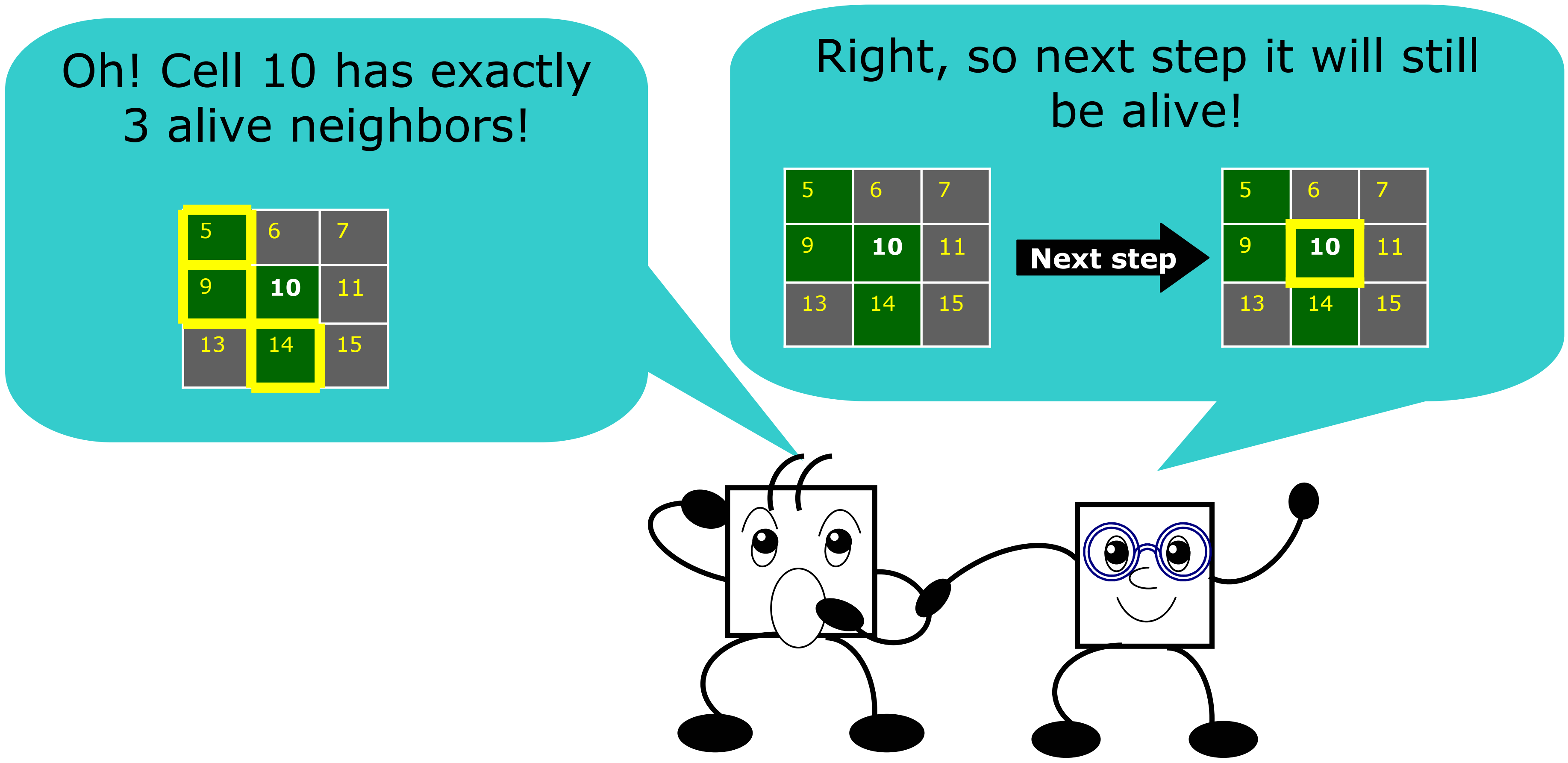


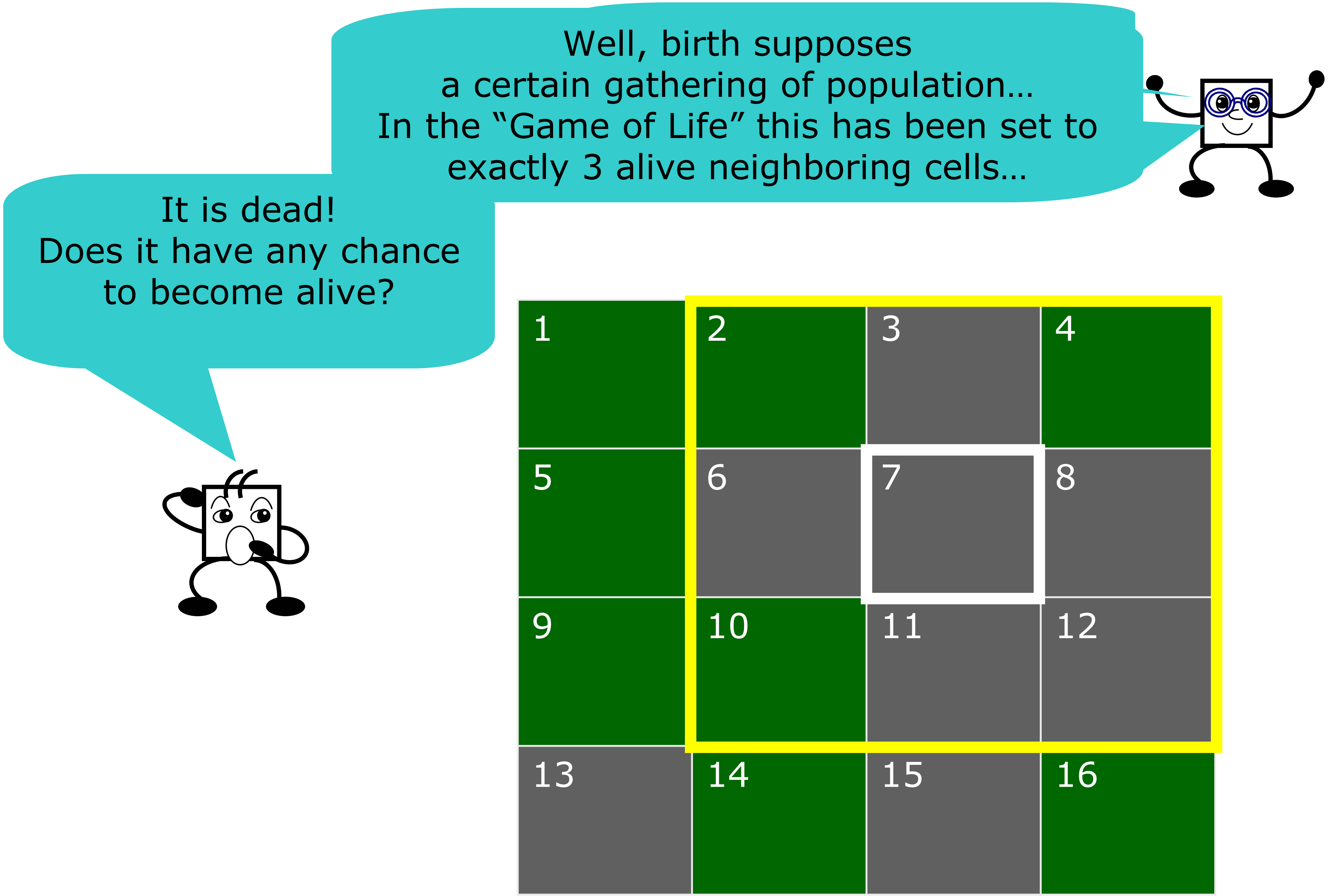
Look at cell #10
and at its 8 neighboring cells

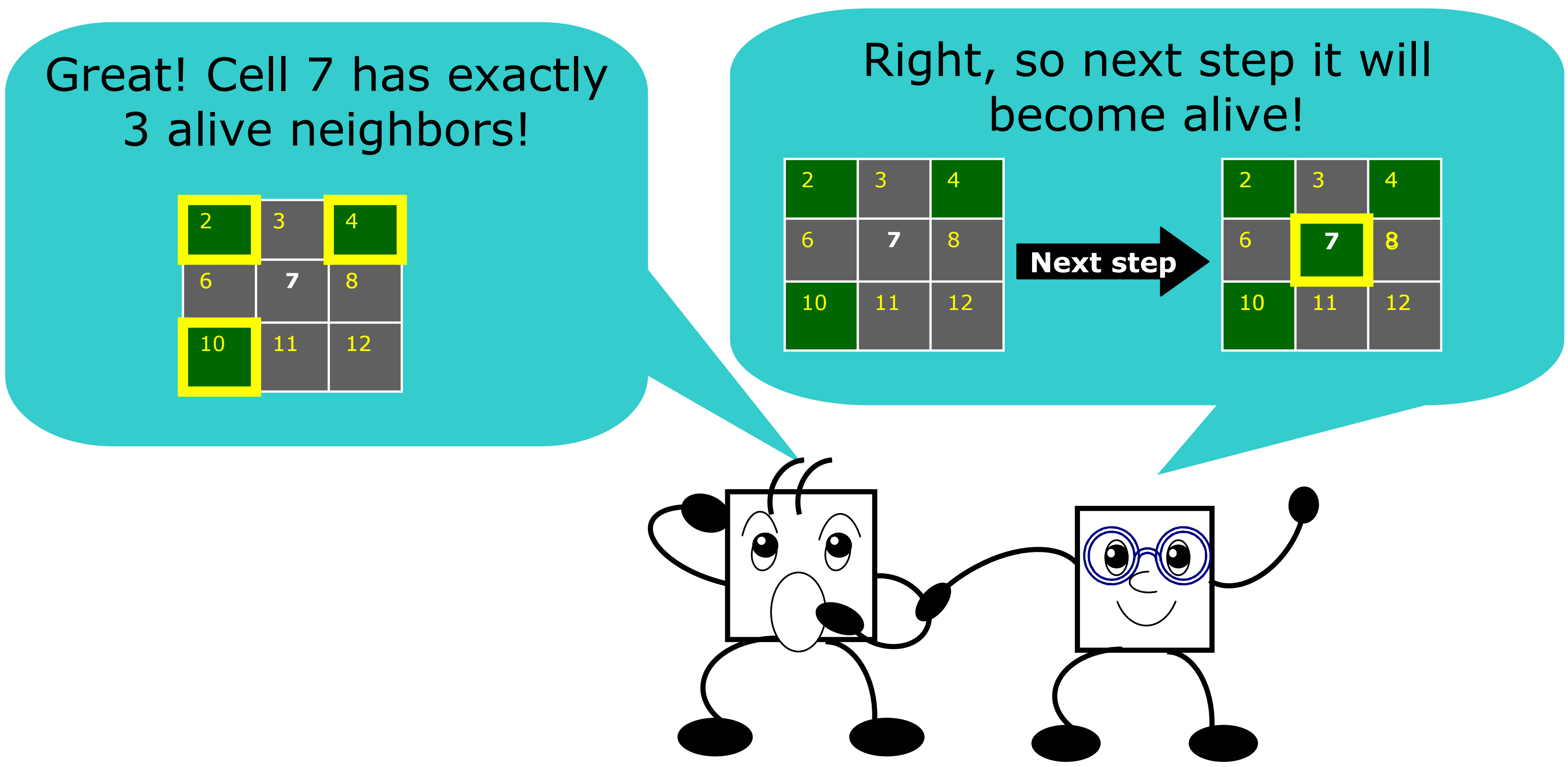




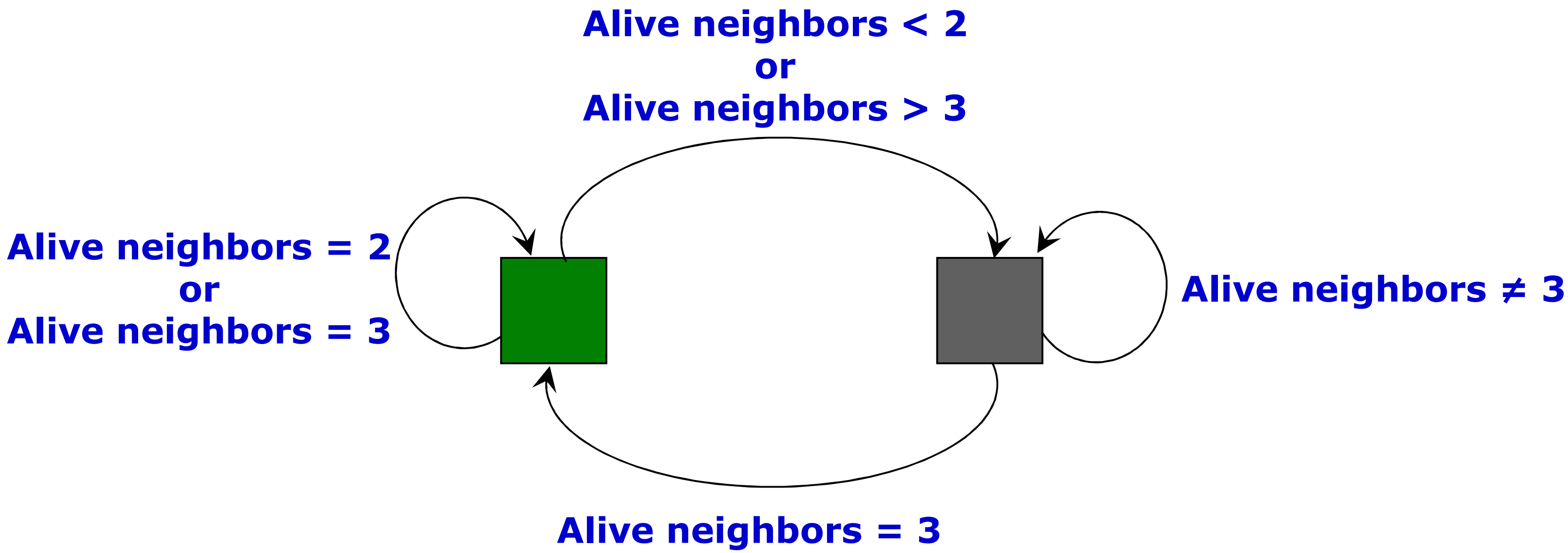
As for any alive cell, we need to consider this rule:
(i) it cannot survive to a too wide isolation
(less than 2 alive neighbors),
(ii) it will also be killed by a too strong concentration
(more than 3 alive neighbors)

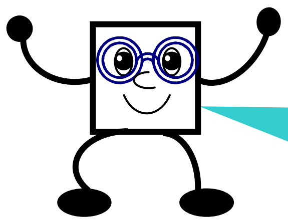

Oh! Cell 10 has exactly 3 alive neighbors!
5 6 7
9 10 11
13 14 15
Right, so next step it will still be alive!
5 6 7
9 10 11
13 14 15
Next step
5 6 7
9 10 11
13 14 15


It is dead!
Does it have any chance
to become alive?
Well, birth supposes
a certain gathering of population…
In the “Game of Life” this has been set to
exactly 3 alive neighboring cells…
1 2 3 4
5 6 7 8
9 10 11 12
13 14 15 16


Great! Cell 7 has exactly 3 alive neighbors!
Right, so next step it will become alive!
Next step

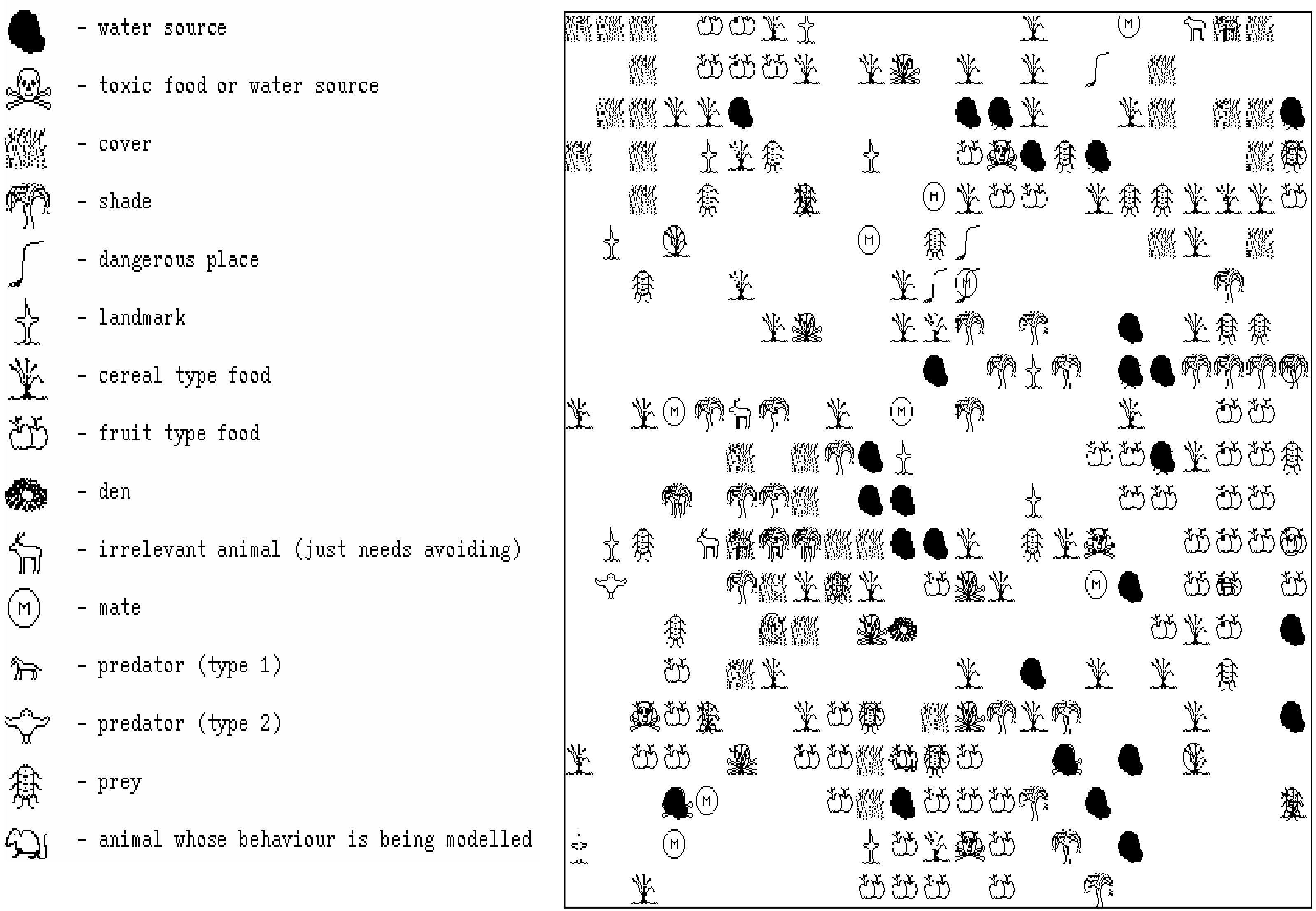
Summary of rules (transition function)





These rules have to be applied
to all cells in the same way
to determine next generation

Individual-based modeling



(from Tyrrell, 1993)

Dog rabies

Mathematical approach
vs simulation-based approach

Transitions between rabies classes
set of 3 coupled, first-order, non-linear differential equations

$$dS/dt = aS - S(\beta I + b + \gamma N) \qquad (1)$$

$$dL/dt = \beta SI - (\sigma + b)L - \gamma NL \qquad (2)$$

$$dI/dt = \sigma L - (\alpha + b)I - \gamma NI \qquad (3)$$

Anderson et al., 1981

Dog rabies
Mathematical approach
vs simulation-based approach

Density of dogs and persistence of rabies

Basic reproduction of rabies in the dog population

$R_0 = \sigma\beta S / ((\sigma + a)(\alpha + a))$ (4)

Threshold density of dogs required for rabies transmission

$K_T = (\sigma + a)(\alpha + a) / \sigma\beta$ (5)

➡ When the density of dogs is below K_T, rabies cannot persist in the population

➡ Vaccination strategies should be related to density of dogs

Kitala et al., 2002

Anderson et al., 1981

Dog rabies dynamics
Mathematical approach
vs simulation-based approach

Vaccination

$$dV/dt = \varphi S - bV - \gamma NV \quad (5)$$

$$dS/dt = a(S + V) - S(\beta I + b + \varphi + \gamma N) \quad (1)$$

Comparison of vaccination rates per year

Comparison of yearly vs twice yearly vaccination coverage

Kitala et al., 2002

Dog rabies dynamics
Mathematical approach
vs simulation-based approach

Rates per year (population) -> simulated events (individuals)

$$dS/dt = a(S + V) - S(\beta I + b + \varphi + \gamma N)$$

$$dL/dt = \beta SI - (\sigma + b)L - \gamma NL$$

$$dI/dt = \sigma L - (\alpha + b)I - \gamma NI$$

$$dV/dt = \varphi S - bV - \gamma NV$$

Spatial heterogeneity

Types of dogs

Home-owned   Community-owned   Stray

Vaccination strategies